Cambridge Assessment International Education

# Cambridge IGCSE™

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**CAMBRIDGE INTERNATIONAL MATHEMATICS** **0607/62**

Paper 6 Investigation and Modelling (Extended) **February/March 2022**

**1 hour 40 minutes**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**
- Answer both part **A** (Questions 1 to 3) and part **B** (Questions 4 to 8).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

**INFORMATION**
- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

Answer **both** parts **A** and **B**.

**A INVESTIGATION (QUESTIONS 1 TO 3)**

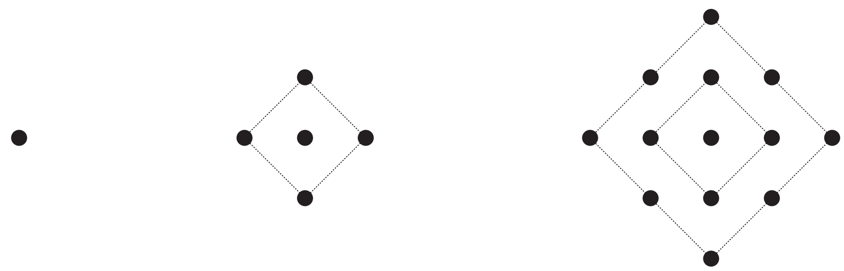
**SEQUENCES OF CENTRED POLYGONS (30 marks)**

You are advised to spend no more than 50 minutes on this part.

This investigation looks at sequences of *centred polygons*.
The first pattern in each sequence is a single dot.
The sequence continues by adding polygons of increasing size around the central dot.

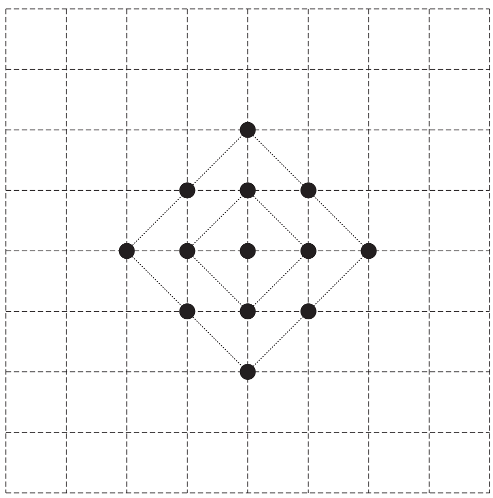
**1 (a)** Oliver draws this sequence of dot patterns called *centred squares*.

Pattern 1 Pattern 2 Pattern 3



**(i)** Pattern 3 is drawn on the grid.

Complete the diagram to show Pattern 4.



[1]

**(ii)** Complete the table.

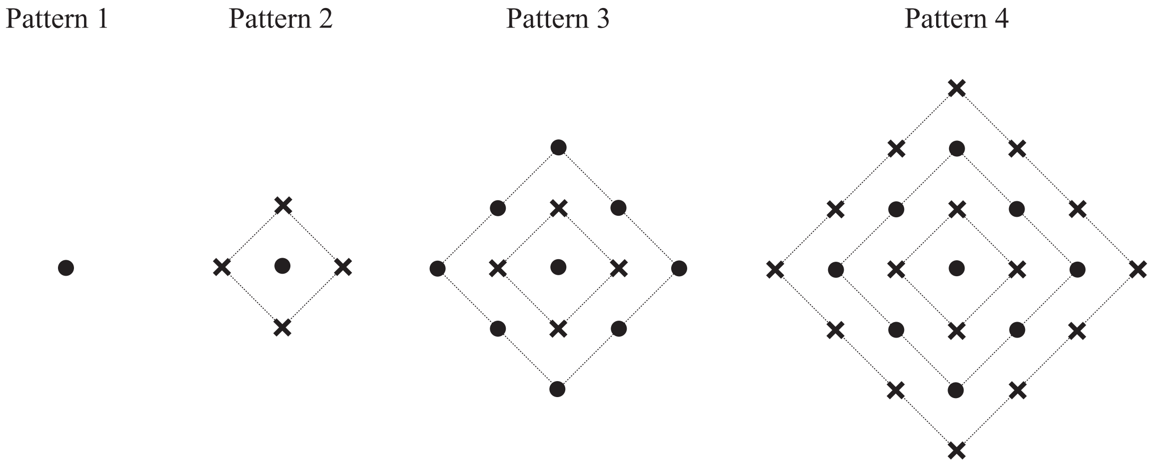
| Pattern number, $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of dots | 1 | 5 | 13 | | |

[2]

**(iii)** Work out the number of dots in Pattern 7.

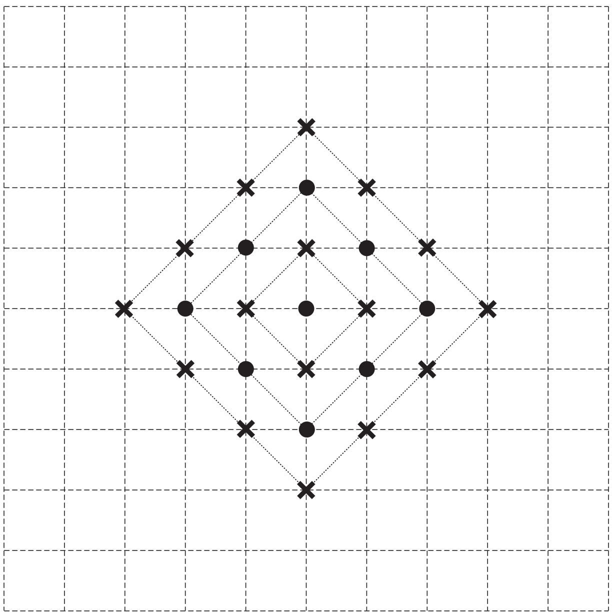
................................................ [2]

**(b)** Oliver draws the dot patterns of centred squares using dots and crosses.




**(i)** Pattern 4 is drawn on the grid.

Complete the diagram to show Pattern 5.



[1]

**(ii)** Complete the table.

| Pattern number, $n$ | Number of dots | Number of crosses | Total number of dots and crosses |
|---|---|---|---|
| 1 | 1 | 0 | 1 |
| 2 | 1 | 4 | 5 |
| 3 | 9 | 4 | 13 |
| 4 | | 16 | |
| 5 | | | |
| 6 | | | |

[2]

**(iii)** Complete the table.

| Pattern number, $n$ | Number of dots | Number of crosses | Total number of dots and crosses |
|---|---|---|---|
| 1 | $1 = 1^2$ | $0 = 0^2$ | $1^2 + 0^2 = 1$ |
| 2 | $1 = 1^2$ | $4 = 2^2$ | $2^2 + 1^2 = 5$ |
| 3 | $9 = 3^2$ | $4 = 2^2$ | $3^2 + 2^2 = 13$ |
| 4 | | $16 =$ | |
| 5 | | | |
| 6 | | | |

[2]

**(iv)** Complete the formula for the total number of dots and crosses, $T$, in Pattern $n$.

$T =$ ................................................ [2]

**(v)** Show that the formula for $T$ is

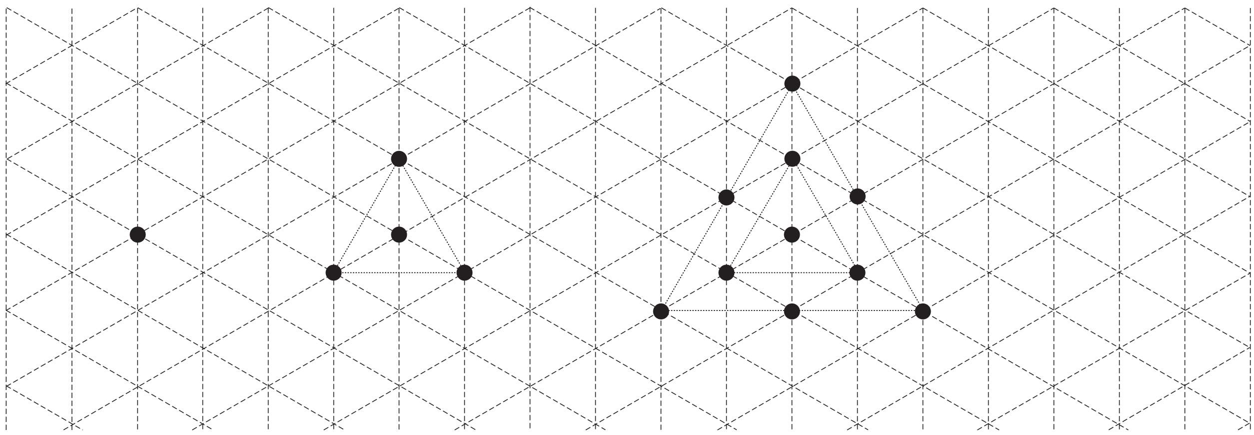
$$2n^2 - 2n + 1.$$

[1]

**(vi)** Work out the total number of dots and crosses in Pattern 15.

................................................ [2]

**2** This is the sequence of centred triangles.




**(a)** Complete the table.

| Pattern number, $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of dots, $T$ | 1 | 4 | 10 | | |

[3]

**(b)** The formula for the number of dots, $T$, in Pattern $n$ is

$$T = an^2 + bn + c\ .$$

**(i)** Give a reason why $a$ is $\frac{3}{2}$.

............................................................................................................................ [1]

**(ii)** Find the value of $b$ and the value of $c$.

$b =$ ................................................

$c =$ ................................................ [4]

**(c)** There are 571 dots in Pattern $k$.

Find the value of $k$.

$k =$ ............................................... [4]

**3** The formula for the number of dots, $T$, in the sequence of centred hexagons is

$$T = 3n^2 - 3n + 1.$$

The number of dots in the $n$th centred hexagon is 6 more than the number of dots in the $n$th centred square.

Find the value of $n$ and the number of dots in the centred hexagon.

$n =$ ................................................

number of dots in the centred hexagon : ................................................ [3]

## B MODELLING (QUESTIONS 4 TO 8)

### DAYLIGHT HOURS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This task looks at modelling the number of daylight hours throughout the year.
The number of daylight hours is the length of time between sunrise and sunset.

**4** Sophia looks at the number of daylight hours in London, England.

She knows the shortest number of daylight hours, *shortest day*, is on 21 December.
She knows the longest number of daylight hours, *longest day*, is on 21 June.

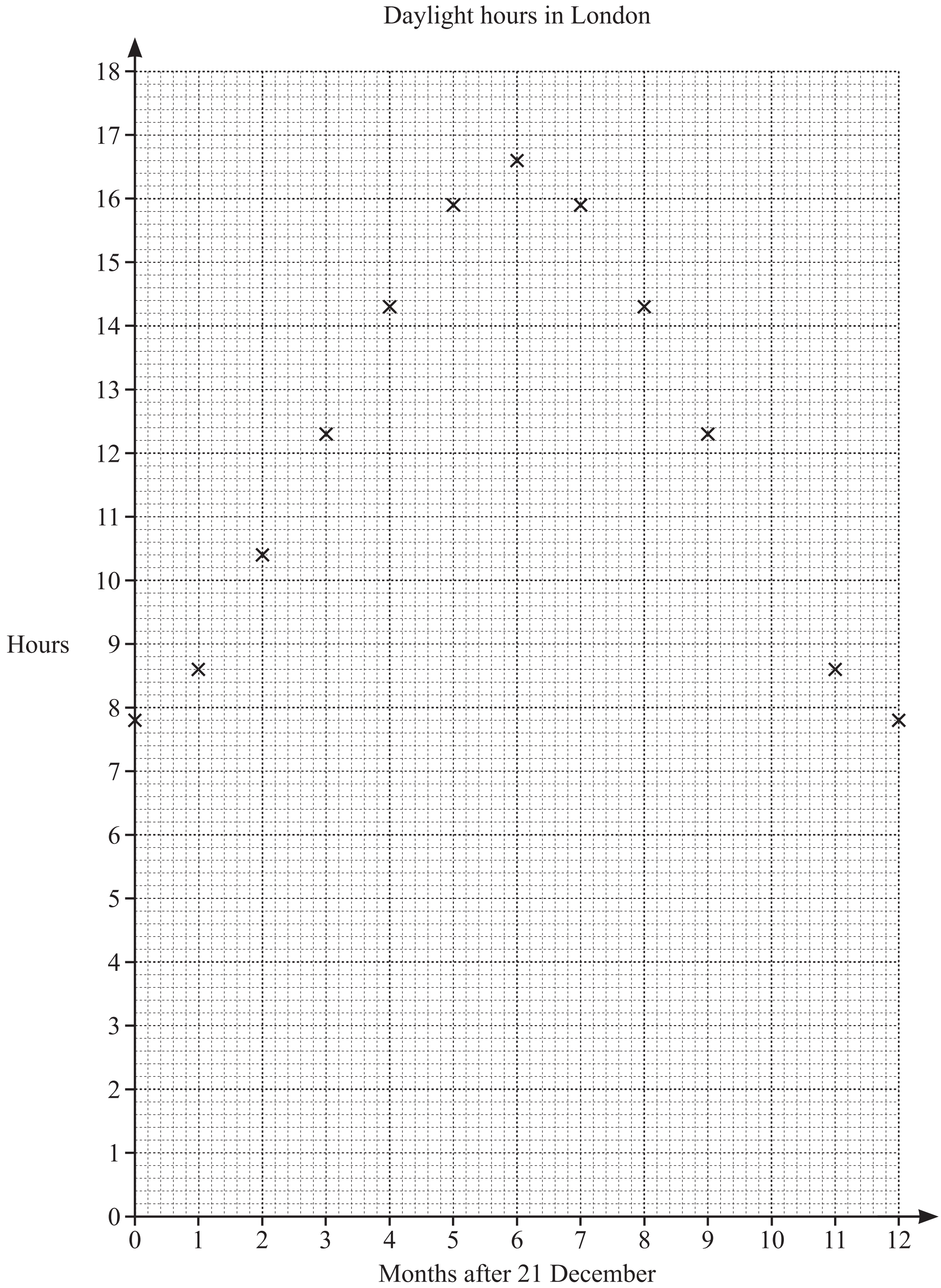
She collects data for the sunrise and sunset times on the 21st day of every month.
She works out the length of daylight in hours and minutes and converts it to a time in hours correct to 1 decimal place.

Her results are in this table.

| Daylight hours in London | | | | | |
|---|---|---|---|---|---|
| Month | Day | Sunrise | Sunset | Daylight in hours and minutes | Daylight in hours |
| December | 21 | 08 03 | 15 53 | 7 h 50 min | 7.8 |
| January | 21 | 07 52 | 16 30 | 8 h 38 min | 8.6 |
| February | 21 | 07 02 | 17 26 | 10 h 24 min | 10.4 |
| March | 21 | 06 00 | 18 15 | 12 h 15 min | 12.3 |
| April | 21 | 05 51 | 20 07 | 14 h 16 min | 14.3 |
| May | 21 | 05 00 | 20 54 | 15 h 54 min | 15.9 |
| June | 21 | 04 43 | 21 21 | 16 h 38 min | 16.6 |
| July | 21 | 05 08 | 21 04 | 15 h 56 min | 15.9 |
| August | 21 | 05 55 | 20 10 | 14 h 15 min | 14.3 |
| September | 21 | 06 45 | 19 00 | 12 h 15 min | 12.3 |
| October | 21 | 07 35 | 17 54 | | |
| November | 21 | 07 28 | 16 03 | 8 h 35 min | 8.6 |
| December | 21 | 08 03 | 15 53 | 7 h 50 min | 7.8 |

**(a)** Complete the row for October. [2]

**(b)** Sophia puts her results from the table on this graph.




**(i)** Plot the value for daylight hours on 21 October. [1]

**(ii)** Write down the number of daylight hours on the shortest day and on the longest day in London.

shortest day ................................................

longest day ................................................ [1]

**(iii)** Show that the length of time halfway between the number of daylight hours on the shortest day and the number of daylight hours on the longest day is 12.2 hours.

[1]

**5** Sophia changes the order of her table.

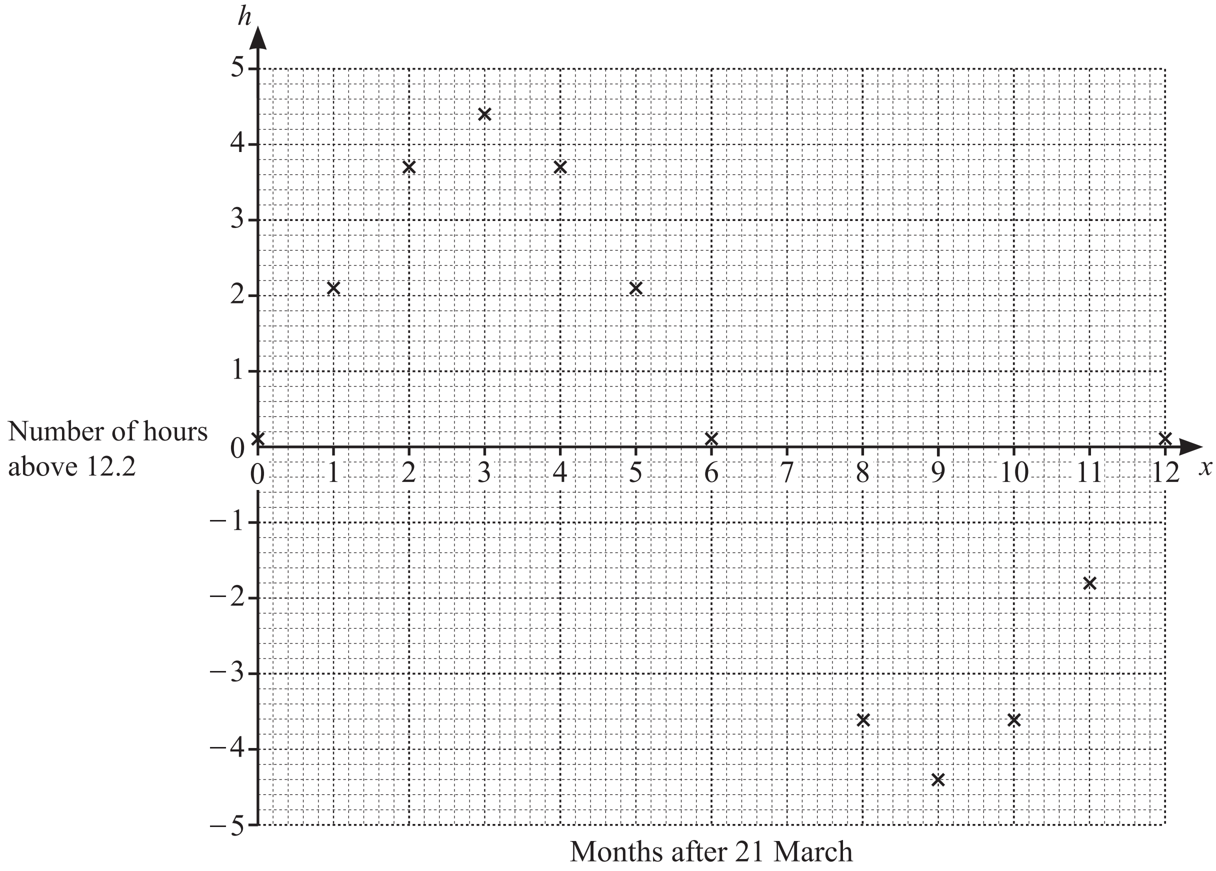
- She starts with 21 March.
- She works out daylight hours above and below 12.2 .

Daylight hours in London

| Month | Day | Sunrise | Sunset | Daylight in hours and minutes | Daylight in hours, $H$ | Daylight in hours above 12.2 , $h$ |
|---|---|---|---|---|---|---|
| March | 21 | 06 00 | 18 15 | 12 h 15 min | 12.3 | 0.1 |
| April | 21 | 05 51 | 20 07 | 14 h 16 min | 14.3 | 2.1 |
| May | 21 | 05 00 | 20 54 | 15 h 54 min | 15.9 | 3.7 |
| June | 21 | 04 43 | 21 21 | 16 h 38 min | 16.6 | 4.4 |
| July | 21 | 05 08 | 21 04 | 15 h 56 min | 15.9 | 3.7 |
| August | 21 | 05 55 | 20 10 | 14 h 15 min | 14.3 | 2.1 |
| September | 21 | 06 45 | 19 00 | 12 h 15 min | 12.3 | 0.1 |
| October | 21 | 07 35 | 17 54 | | | |
| November | 21 | 07 28 | 16 03 | 8 h 35 min | 8.6 | − 3.6 |
| December | 21 | 08 03 | 15 53 | 7 h 50 min | 7.8 | − 4.4 |
| January | 21 | 07 52 | 16 30 | 8 h 38 min | 8.6 | − 3.6 |
| February | 21 | 07 02 | 17 26 | 10 h 24 min | 10.4 | − 1.8 |
| March | 21 | 06 00 | 18 15 | 12 h 15 min | 12.3 | 0.1 |

**(a)** Complete the row for October. [1]

**(b)** Sophia puts her results from the table into this graph.




Sophia models the number of daylight hours in London using the points on her graph. She decides on this model.

$$h = 4.4\sin(30x)$$

$h$ is the number of daylight hours above 12.2 and $x$ is the number of months after 21 March.

Give a reason why Sophia uses

**(i)** the number 4.4,

.................................................................................................................................

................................................................................................................................. [1]

**(ii)** the number 30.

.................................................................................................................................

................................................................................................................................. [1]

**(c)** Sophia changes her model.
$H$ is the number of daylight hours and $x$ is the number of months after 21 March.

Complete the model for $H$.

$H =$ ............................................... [1]

**(d)** **(i)** Use the model in **part (c)** to calculate the number of daylight hours on 21 April and 21 January.

April ...............................................

January ............................................... [4]

**(ii)** Sophia thinks that the calculated values for April and January show that her model is a good one.

Is she correct?
Give a reason for your answer.

.............................................................................................................................

............................................................................................................................. [1]

**(iii)** What assumption has Sophia made about the months for her model?

............................................................................................................................. [1]

**6** This table shows sunrise and sunset times for Tokyo in Japan on the 21st day of every month.

| Sunrise and sunset times in Tokyo | | | |
|---|---|---|---|
| Month | Day | Sunrise | Sunset |
| March | 21 | 05 43 | 17 53 |
| April | 21 | 05 00 | 18 19 |
| May | 21 | 04 32 | 18 44 |
| June | 21 | 04 25 | 19 00 |
| July | 21 | 04 40 | 18 53 |
| August | 21 | 05 04 | 18 23 |
| September | 21 | 05 28 | 17 39 |
| October | 21 | 05 52 | 16 58 |
| November | 21 | 06 22 | 16 30 |
| December | 21 | 06 47 | 16 32 |
| January | 21 | 06 48 | 16 56 |
| February | 21 | 06 22 | 17 27 |
| March | 21 | 05 43 | 17 53 |

The shortest day is 21 December and the longest day is 21 June.

Find a model for the number of daylight hours, $H$, in Tokyo.
Write your model in the form $H = A\sin(Bx) + C$,
where $x$ is the number of months after 21 March and $A$, $B$ and $C$ are numbers to be found.

$H =$ ............................................... [6]

**7** This is a model of the number of daylight hours in Cairo in Egypt.

$$H = 2\sin(30x) + 12.2$$

$H$ is the number of daylight hours and $x$ is the number of months after 21 March.

**(a)** Use the model to estimate the number of daylight hours on the shortest day in Cairo.

................................................. [2]

**(b)** What is the date of the shortest day in Cairo according to the model?

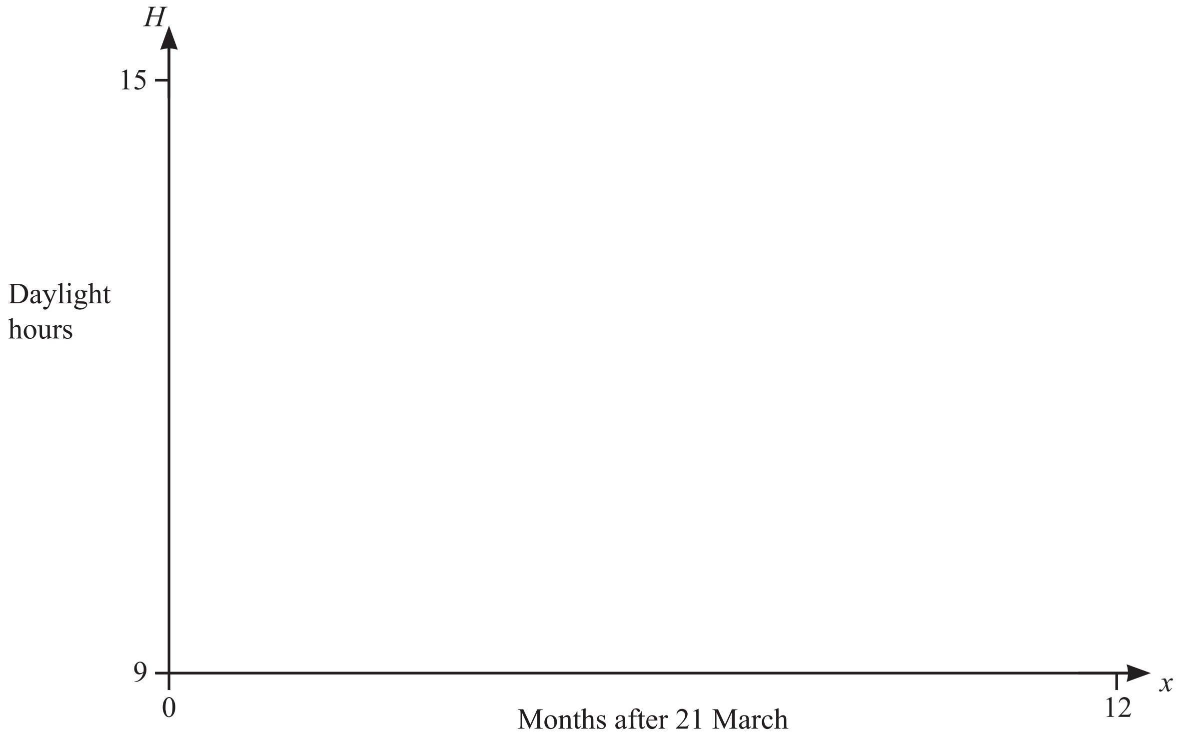
................................................. [1]

**8** This is a model of the number of daylight hours in Melbourne in Australia.

$$H = 2.4\sin(30x + 180) + 12.2$$

$H$ is the number of daylight hours and $x$ is the number of months after 21 March.

**(a)** On the axes below sketch the models for the number of daylight hours:
- in Cairo
- in Melbourne.




[3]

**(b)** Make a statement about the number of daylight hours in Melbourne on the shortest day in Cairo.

.......................................................................................................................................... [1]

**(c)** Find the dates when the number of daylight hours in Cairo is the same as it is in Melbourne.

.......................................................................................................................................... [2]

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