Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

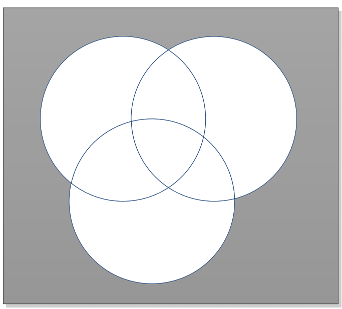
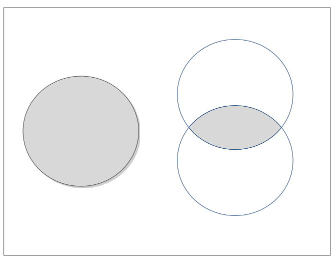
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

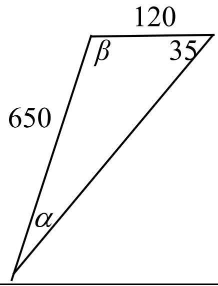
awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1(a)	$\mathscr{E}$ 	**B1**	
	$\mathscr{E}$ 	**B1**	

Question	Answer	Marks	Guidance
1(b)	$P = \{30^\circ, 150^\circ, 210^\circ, 330^\circ\}$	B1	May be seen or implied in a Venn diagram Allow without set notation
	$Q = \{30^\circ, 150^\circ\}$	B1	May be seen or implied in a Venn diagram Allow without set notation
	$P \cap Q = \{30^\circ, 150^\circ\}$	B1	**Dep** on both previous B marks Must be in set notation
2	**Either:** $(2x+3)^2(x-1) = 3(2x+3)$ $(2x+3)(2x^2+x-6)(=0)$	M1	For attempt to equate line and curve and attempt to simplify to $2x+3$ × a quadratic factor or cancelling $2x+3$ and obtaining a quadratic factor
	$(2x+3)(2x^2+x-6)=0$ $(2x+3)(2x-3)(x+2)=0$	M1	**Dep** for attempt at 3 linear factors from a linear term and a quadratic term
	$\left(-\frac{3}{2}, 0\right)$	B1	
	$\left(\frac{3}{2}, 18\right)$	A1	**Dep** on first M mark only
	$(-2, -3)$	A1	**Dep** on first M mark only
	Or: $(2x+3)^2(x-1) = 3(2x+3)$ $4x^3+8x^2-9x-18(=0)$	M1	For attempt to equate line and curve and attempt to simplify to a cubic equation, by collecting like terms
	$(x+2)(4x^2-9)$ $(2x-3)(2x^2+7x+6)$ $(2x+3)(2x^2+x-6)$ $(2x+3)(2x-3)(x+2)(=0)$	M1	**Dep** For attempt to find a factor from a 4 term cubic equation (usually $x+2$), do long division oe to obtain a quadratic factor and factorise this quadratic factor
	$\left(-\frac{3}{2}, 0\right)$	A1	
	$\left(\frac{3}{2}, 18\right)$	A1	
	$(-2, -3)$	A1	
3(i)	1000	B1	

Question	Answer	Marks	Guidance
3(ii)	$\dfrac{dB}{dt} = 400e^{2t} - 1600e^{-2t}$	B1	
	$3 = e^{2t} - 4e^{-2t}$ oe	M1	For equating an equation of the form $ae^{2t} + be^{-2t}$ to 1200 and dividing by 400
	$e^{4t} - 3e^{2t} - 4 = 0$	A1	
3(iii)	$\left(e^{2t}+1\right)\left(e^{2t}-4\right) = 0$	M1	For attempt to factorise and solve, dealing with exponential correctly, to obtain $e^{2t} = \ldots$
	$t = \ln 2,\ \dfrac{1}{2}\ln 4$ or awrt 0.693 only isw if appropriate	A1	
4(a)	$a = \dfrac{5}{2}$	B1	
	$b = -\dfrac{3}{2}$	B1	
	$c = \dfrac{11}{2}$	B1	
4(b)	$9x^{\frac{1}{2}} - 3y^{-\frac{1}{2}} = 12$ $4x^{\frac{1}{2}} + 3y^{-\frac{1}{2}} = 14$	M1	For attempt to solve simultaneous equations. Must reach $kx^{\frac{1}{2}} = \ldots$ or $ky^{-\frac{1}{2}} = \ldots$ oe
	$x = 4$	A1	
	$y = \dfrac{1}{4}$	A1	
5(i)	$9.6 = 12\theta$	M1	For use of arc length
	$\theta = 0.8$	A1	

Question	Answer	Marks	Guidance
5(ii)	Either $\tan\theta = \frac{AB}{12}$, $(AB = 12.36)$ Or $OB = \frac{12}{\cos\theta}$ $(OB = 17.22)$	**M1**	For attempt to find AB or OB using *their* θ May be implied by a correct triangle area Allow if using degrees consistently
	Either Area $\Delta OAB = \frac{1}{2}\times 12\times$ *their* 12.36 Or Area $\Delta OAB = \frac{1}{2}\times 12\times$ *their* $17.22\times\sin\theta$ $(= 74.1$ or $74.2)$	**M1**	Allow if using degrees consistently For attempt to find area of triangle using *their* θ
	Area of sector $OAC = \frac{1}{2}\times 12^2\times 0.8$ $= 57.6$	**B1**	Allow unsimplified
	Area of shaded region = 16.5 or 16.6	**A1**	
6(a)(i)	40320	**B1**	
6(a)(ii)	No. of ways with maths books as 1 unit = $5!$ or $5\times 4!$ or ${}^5P_5$ or 120	**B1**	
	No. of ways maths books can be arranged amongst themselves = 4! or ${}^4P_4$ or 24	**B1**	
	Total = $(5!\ \times 4!$ oe$) = 2880$	**B1**	
6(a)(iii)	No. of ways with maths books as 1 unit and geography books as 1 unit = $3!$ or ${}^3P_3$ or $3\times 2!$ or 6	**B1**	
	No. of ways maths books can be arranged amongst themselves and geography books can be arranged amongst themselves $= 4!\ \times 3!$ or ${}^4P_4\times{}^3P_3$ or 144	**B1**	
	Total = $(3!\ \times 4!\ \times 3!$ oe$)$ = 864	**B1**	
6(b)(i)	${}^{12}C_6 = 924$	**B1**	

Question	Answer	Marks	Guidance
6(b)(ii)	**Either:** $924 - {}^8C_6$	**M1**	For *their* **(i)** – the number of teams of just men
	Total = 896	**A1**	
	Or: 5M 1W : ${}^8C_5 \times {}^4C_1$ $(=224)$ 4M 2W : ${}^8C_4 \times {}^4C_2$ $(=420)$ 3M 3W : ${}^8C_3 \times {}^4C_3$ $(=224)$ 2M 4W : ${}^8C_2 \times {}^4C_4$ $(=28)$	**M1**	For a complete method
	Total = 896	**A1**	
7(i)		**B1**	For correct triangle, may be implied by a correct sine rule or cosine rule.
	$\frac{120}{\sin\alpha}$ or $\frac{120}{\sin(55-\theta)} = \frac{650}{\sin 35}$ or $\frac{650}{\sin 145}$	**M1**	For use of a correct sine rule to obtain $\alpha = \ldots$ or $\theta = \ldots$ Or for a correct cosine rule leading to a value for *v*, followed by a correct sine rule leading to one of the other angles
	$\alpha = 6.08^\circ$ or $\beta = 138.9$	**A1**	May be implied by a correct $\theta =$ awrt 49°
	Bearing is 048.9° or 049°	**A1**	
7(ii)	Either $\frac{v_r}{\sin(145 - their\alpha)} = \frac{650}{\sin 35}$ or $\frac{120}{\sin(their\alpha)}$ Or $v_r^2 = 650^2 + 120^2 -$ $(2 \times 650 \times 120)\cos(145 - their\alpha)$	**M1**	For use of sine rule or cosine rule to find resultant velocity Do not allow for a right-angled triangle May be seen in **(i)**
	$v_r = 745$	**A1**	For correct resultant velocity, allow awrt 745
	Time taken = $\frac{1250}{their\ 744.7}$	**M1**	For correct attempt at finding time using *their v* , $\neq 650$, 120, 770 or 530
	$= 1.68$ hours or I hour 41 mins or 101 mins	**A1**	

Question	Answer	Marks	Guidance
8(i)	$e^y = \frac{m}{x} + c$	B1	May be implied by subsequent work
	Either $20 = 2m + c$ $8 = 4m + c$	M1	For at least 1 correct equation
		M1	**Dep** For attempt to solve *their* 2 equations simultaneously to obtain at least one unknown
	leading to $m = -6,\ c = 32$	A1	For both
	$y = \ln\left(32 - \frac{6}{x}\right)$	A1	Must have correct brackets Mark the final answer given
	Or: Gradient = $m = (-6)$	M1	For attempt to find gradient and equate it to *m*
	$20 = 2m + c$ or $8 = 4m + c$ or $e^y - 8 = m\left(\frac{1}{x} - 4\right)$ or $e^y - 20 = m\left(\frac{1}{x} - 2\right)$	M1	For at least 1 correct equation, may be using *their m*
	leading to $c = 32$ and $m = -6$	A1	For both $m = -6,\ c = 32$
	$y = \ln\left(32 - \frac{6}{x}\right)$	A1	
8(ii)	$x > \frac{3}{16}$ oe	B1	
8(iii)	$y = \ln 30$ isw	B1	
8(iv)	$2 = \ln\left(32 - \frac{6}{x}\right)$	M1	For a correct substitution and attempt to re-arrange using 2, *their* 32 and *their* – 6, keeping exactness to obtain $x =$
	$x = \frac{6}{32 - e^2}$ oe	A1	Must be exact

Question	Answer	Marks	Guidance
9(i)	$5 = 4 + 2\cos 3x$	**M1**	For attempt to solve trig equation to obtain one correct solution
	$\frac{\pi}{9}$	**A1**	
	$-\frac{\pi}{9}$	**A1**	
9(ii)	**Either:** $\int_{-\frac{\pi}{9}}^{\frac{\pi}{9}} 4 + 2\cos 3x - 5 \, dx$	**M1**	For use of subtraction method
	$\left[\frac{2}{3}\sin 3x - x\right]_{-\frac{\pi}{9}}^{\frac{\pi}{9}}$	**M1**	For attempt to integrate to obtain the form $a\sin 3x + bx$
		B1	For $\frac{2}{3}\sin 3x$
		B1	For $-x$, may be implied by $4x - 5x$
	$\left(\frac{\sqrt{3}}{3} - \frac{\pi}{9}\right) - \left(-\frac{\sqrt{3}}{3} + \frac{\pi}{9}\right)$	**M1**	**Dep** on previous M mark for correct application of *their* limits in radians from **(i)** retaining exactness
	Shaded area $= \frac{2\sqrt{3}}{3} - \frac{2\pi}{9}$ oe isw	**A1**	

Question	Answer	Marks	Guidance
9(ii)	**Or:** Area of rectangle = $5\times\frac{2\pi}{9}$	M1	$5\times$ the difference of *their* limits in exact radians
	Area under curve = $\left[4x+\frac{2}{3}\sin 3x\right]_{-\frac{\pi}{9}}^{\frac{\pi}{9}}$	M1	For attempt to integrate to obtain the form $a\sin 3x+bx$
		B1	For $\frac{2}{3}\sin 3x$
		B1	For $4x$
	$\left(\frac{\sqrt{3}}{3}+\frac{4\pi}{9}\right)-\left(-\frac{\sqrt{3}}{3}-\frac{4\pi}{9}\right)$ $\left(=\frac{2\sqrt{3}}{3}+\frac{8\pi}{9}\right)$	M1	**Dep** on previous M mark for correct application of *their* limits in exact radians from **(i)** retaining exactness
	Shaded area = $\frac{2\sqrt{3}}{3}-\frac{2\pi}{9}$ oe isw	A1	
10(i)	$800=4x^2h$	B1	
	$h=\frac{800}{4x^2}$ oe or $xh=\frac{800}{4x}$ oe	B1	
	$(S=)2hx+8xh+4x^2$ oe	M1	Allow if h is substituted at this point
	$S=4x^2+\left(\frac{2000}{x}\right)$	A1	Leading to AG, must have S = or surface area = at some point and no errors

Question	Answer	Marks	Guidance
10(ii)	$\left(\frac{dS}{dx}=\right)8x-\frac{2000}{x^2}$	**B1**	For correct differentiation
	When $\frac{dS}{dx}=0$, $x=\sqrt[3]{250}$ oe (6.30)	**M1**	For equating to zero and attempt to solve, must get as far as $x = \ldots$, must be using the form $ax+\frac{b}{x^2}$
		A1	For correct positive x
	$S=476$ only	**A1**	
	$\frac{d^2S}{dx^2}=8+\frac{4000}{x^3}$ $\frac{d^2S}{dx^2}>0$ or 24 so minimum	**B1**	For a correct convincing method, with enough detail to reach a correct conclusion of a minimum. Must be using $x=\sqrt[3]{250}$ oe
11		**M1**	For attempt at differentiating a product
		B1	For $\frac{2}{3}\times3(3x+1)^{-\frac{1}{3}}$
	$\left(\frac{dy}{dx}=\right)(x-2)\times\frac{2}{3}\times3(3x+1)^{-\frac{1}{3}}+(3x+1)^{\frac{2}{3}}$	**A1**	For all other terms correct
	$y=\frac{4}{3}$	**B1**	
	When $x=\frac{7}{3}$, $\frac{dy}{dx}=\frac{13}{3}$	**M1**	For attempt at normal equation using $-\frac{1}{their\ m}$ and *their y* when $x=\frac{7}{3}$
	Equation of normal: $y-\frac{4}{3}=-\frac{3}{13}\left(x-\frac{7}{3}\right)$	**A1**	For correct normal equation, may be implied by a correct final answer
	At y-axis, $y=\frac{73}{39}$ $\left(0,\frac{73}{39}\right)$ isw	**A1**	