Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

1052433858

ADDITIONAL MATHEMATICS **4037/11**

Paper 1 **May/June 2023**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

DC (ST) 326645

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

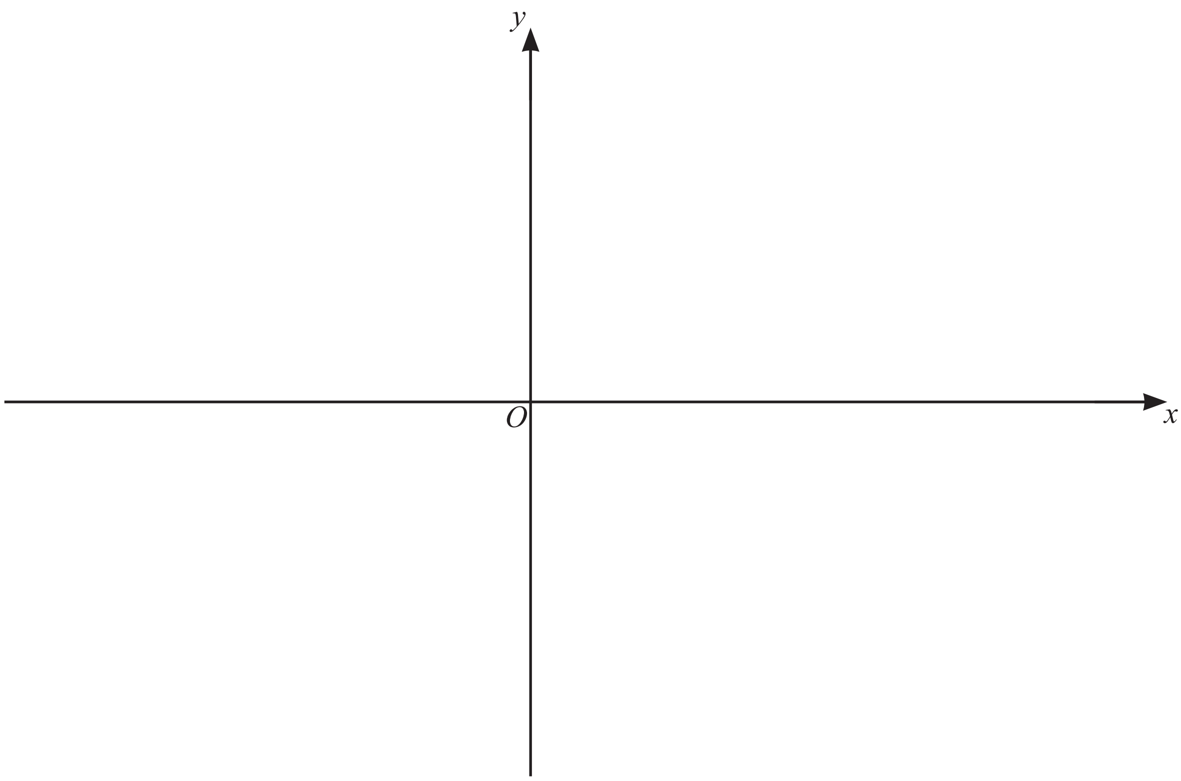
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Write $5x^2 - 14x + 8$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be found. [3]

(b) Hence write down the coordinates of the stationary point on the curve $y = 5x^2 - 14x + 8$. [2]

(c) On the axes below, sketch the graph of $y = \left|5x^2 - 14x + 8\right|$, stating the coordinates of the points where the graph meets the coordinate axes. [3]



(d) Write down the range of values of k for which the equation $\left|5x^2 - 14x + 8\right| = k$ has 4 distinct roots. [2]

2 The polynomial p is such that $p(x) = ax^3 + 7x^2 + bx + c$, where a, b and c are integers.

(a) Given that $p''\left(\frac{1}{2}\right) = 32$, show that $a = 6$. [2]

(b) Given that $p(x)$ has a factor of $3x - 4$ and a remainder of 7 when divided by $x + 1$, find the values of b and c. [4]

(c) Write $p(x)$ in the form $(3x-4)q(x)$, where $q(x)$ is a quadratic factor. [2]

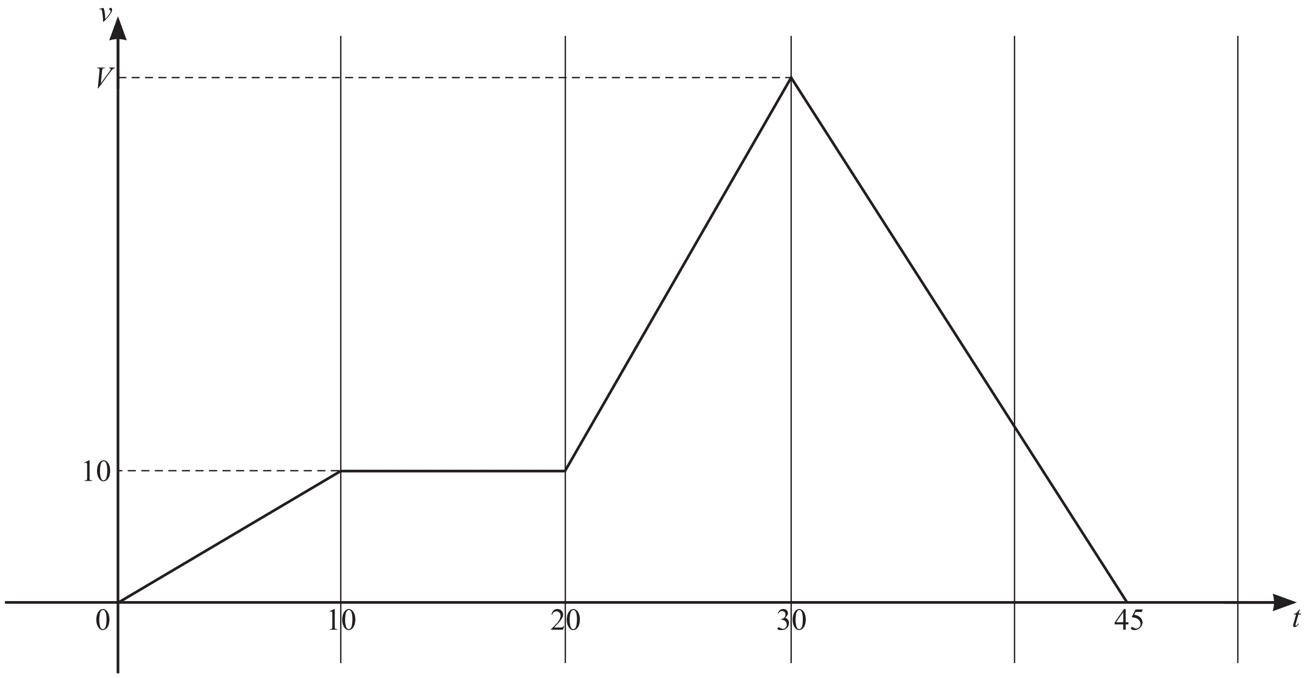
(d) Hence write $p(x)$ as a product of linear factors with integer coefficients. [1]

3 The points A and B have coordinates $(2, 5)$ and $(10, -15)$ respectively. The point P lies on the perpendicular bisector of the line AB. The y-coordinate of P is -9.

(a) Find the x-coordinate of P. [5]

(b) The point R is the reflection of P in the line AB. Find the coordinates of R. [2]

4



The diagram shows the velocity–time graph for a particle travelling in a straight line with velocity, $v\,\mathrm{ms}^{-1}$, at time t seconds. When $t = 30$ the velocity of the particle is $V\,\mathrm{ms}^{-1}$. The particle travels 800 metres in 45 seconds.

(a) Find the value of V. [2]

(b) Find the acceleration of the particle when $t = 35$. [2]

5 DO NOT USE A CALCULATOR IN THIS QUESTION.

In this question, all lengths are in centimetres.

(a) You are given that $\cos 120° = -\frac{1}{2}$, $\sin 120° = \frac{\sqrt{3}}{2}$ and $\tan 120° = -\sqrt{3}$.

In the triangle ABC, $AB = 5\sqrt{3} - 6$, $BC = 5\sqrt{3} + 6$ and angle $ABC = 120°$. Find AC, giving your answer in the form $a\sqrt{b}$ where a and b are integers greater than 1. [4]

(b) You are given that $\cos 30° = \frac{\sqrt{3}}{2}$, $\sin 30° = \frac{1}{2}$ and $\tan 30° = \frac{1}{\sqrt{3}}$.

In the triangle PQR, $PQ = 3 + 2\sqrt{5}$ and angle $PQR = 30°$. Given that the area of this triangle is $\frac{2 + 5\sqrt{5}}{4}$, find QR, giving your answer in the form $c + d\sqrt{5}$, where c and d are integers. [4]

6 **(a)** Show that $\dfrac{\cot\theta + \tan\theta}{\sec\theta} = \operatorname{cosec}\theta$. [4]

(b) Hence solve the equation $\left(\dfrac{\cot\dfrac{\phi}{3}+\tan\dfrac{\phi}{3}}{\sec\dfrac{\phi}{3}}\right)^2 = 2$, for $-540° < \phi < 540°$. [6]

7 **(a)** A team of 8 people is to be chosen from a group of 15 people.

(i) Find the number of different teams that can be chosen. [1]

(ii) Find the number of different teams that can be chosen if the group of 15 people contains a family of 4 people who must be kept together. [3]

(b) Given that $(n+9) \times {}^{n}\mathrm{P}_{10} = (n^2+243) \times {}^{n-1}\mathrm{P}_9$, find the value of n. [3]

8 A curve has the equation $y = \dfrac{(3x-4)^{\frac{1}{3}}}{2x+1}$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{Ax+B}{(2x+1)^2(3x-4)^{\frac{2}{3}}}$, where A and B are integers to be found. [5]

(b) Find the coordinates of the stationary point on the curve. [2]

9 **(a)** The first three terms of an arithmetic progression are $\ln q$, $\ln q^4$ and $\ln q^7$, where q is a positive constant. The sum to n terms of this progression is $4845 \ln q$. Find the value of n. [3]

(b) The first three terms of a geometric progression are p^{3x}, p^{x} and p^{-x}, where p is a positive integer. Find the nth term of this progression giving your answer in the form $p^{(a+bn)x}$. [3]

(c) The first three terms of a different geometric progression are $\frac{4}{3}\cos^2 3\theta$, $\frac{16}{9}\cos^4 3\theta$ and $\frac{64}{27}\cos^6 3\theta$, for $0 < \theta < \frac{\pi}{3}$. Find the set of values of θ for which this progression has a sum to infinity. [5]

Question 10 is printed on the next page.

10 It is given that $y = (3x+1)^2 \ln(3x+1)$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Hence find $\int (3x+1)\ln(3x+1)\,dx$. [4]