Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1517906375

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2023**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

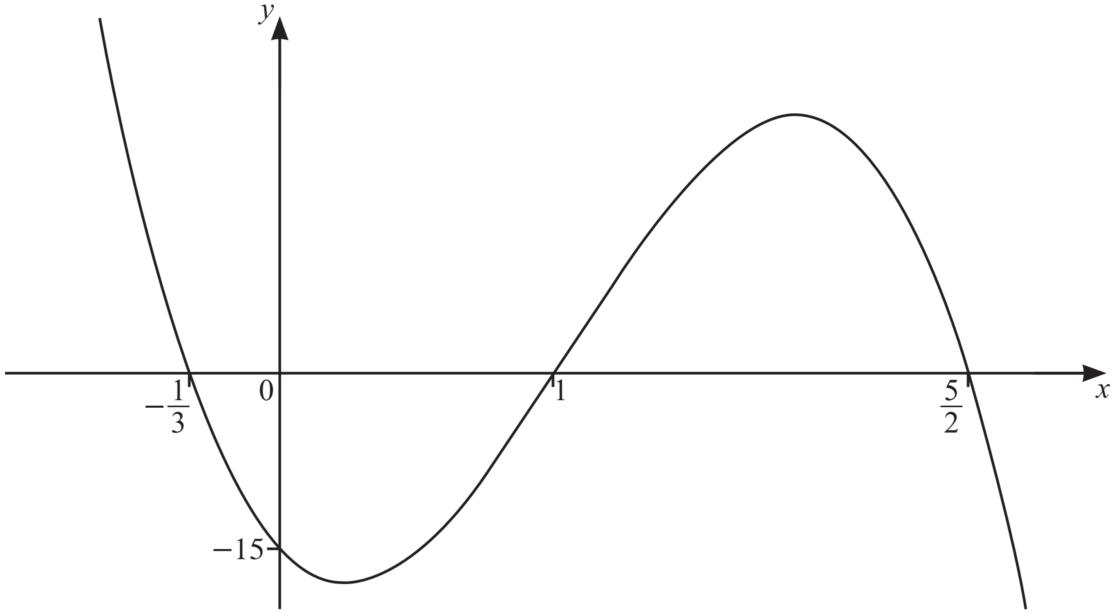
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1



The diagram shows the graph of the cubic polynomial $y = \mathrm{f}(x)$.

(a) Find an expression for $\mathrm{f}(x)$ in factorised form. Write each linear factor with its coefficients as integers. [3]

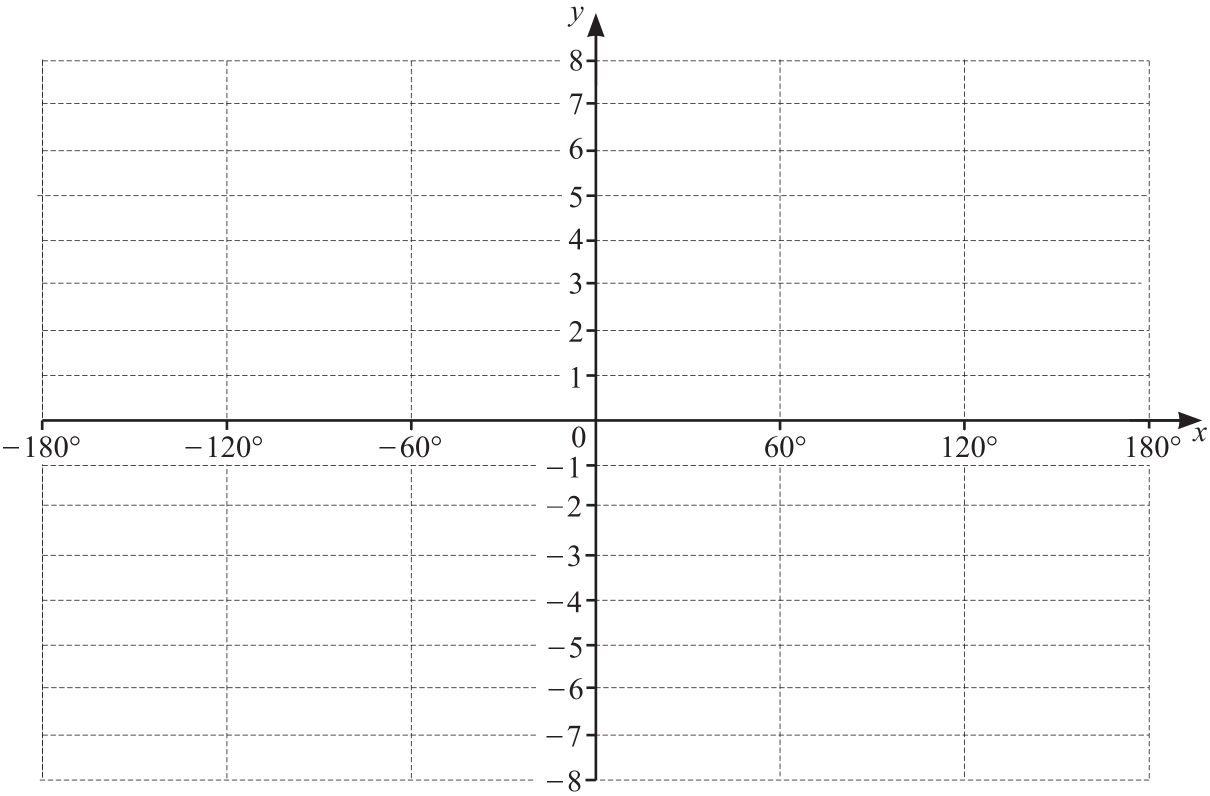
(b) Write down the values of x such that $\mathrm{f}(x) < 0$. [2]

2 The function g is defined by $\mathrm{g}(x) = 5\sin\frac{3x}{4} - 2$ for all values of x.

(a) Write down the amplitude of g. [1]

(b) Write down the period of g in degrees. [1]

(c) On the axes, sketch the graph of $y = \mathrm{g}(x)$, for $-180° \leqslant x \leqslant 180°$. [3]



3 When $\ln(y+2)$ is plotted against x^2 a straight line graph is obtained. The line passes through the points $(2.25, 9.37)$ and $(4.75, 3.92)$. Find y in terms of x. [5]

4 **(a)** It is given that the first four terms, in ascending powers of x, in the expansion of $\left(1-\frac{x}{2}\right)^n$ can be written in the form $1-8x+px^2+qx^3$, where n, p and q are integers. Find the values of n, p and q. [5]

(b) Find the term independent of x in the expansion of $\left(\frac{2}{x^2}+\frac{x}{3}\right)^6$, giving your answer as a rational number. [2]

5 Solve the equation $3\sec^2\left(2\theta + \frac{\pi}{6}\right) = 4$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, giving your answers in terms of π. [5]

6 The polynomial $p(x)$ is such that $p(x) = ax^3 + bx^2 + cx - 5$, where a, b and c are integers. It is given that $p'(0) = 12$. It is also given that $p(x)$ has a factor of $3x - 1$ and a remainder of 95 when divided by $x - 2$.

(a) Find the values of a, b and c. [7]

(b) Show that the equation $p(x) = 0$ has only one real root. [3]

7 **(a)** A 6-digit number is to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit can be used only once in any 6-digit number. A 6-digit number cannot start with 0.

(i) Find how many 6-digit numbers can be formed. [1]

(ii) Find how many of these 6-digit numbers are divisible by 5. [3]

(b) A committee of 7 people is to be chosen from 6 doctors, 10 nurses and 8 dentists.

(i) Find the number of committees that can be chosen. [1]

(ii) Find the number of committees that can be chosen if all the doctors have to be on the committee. [1]

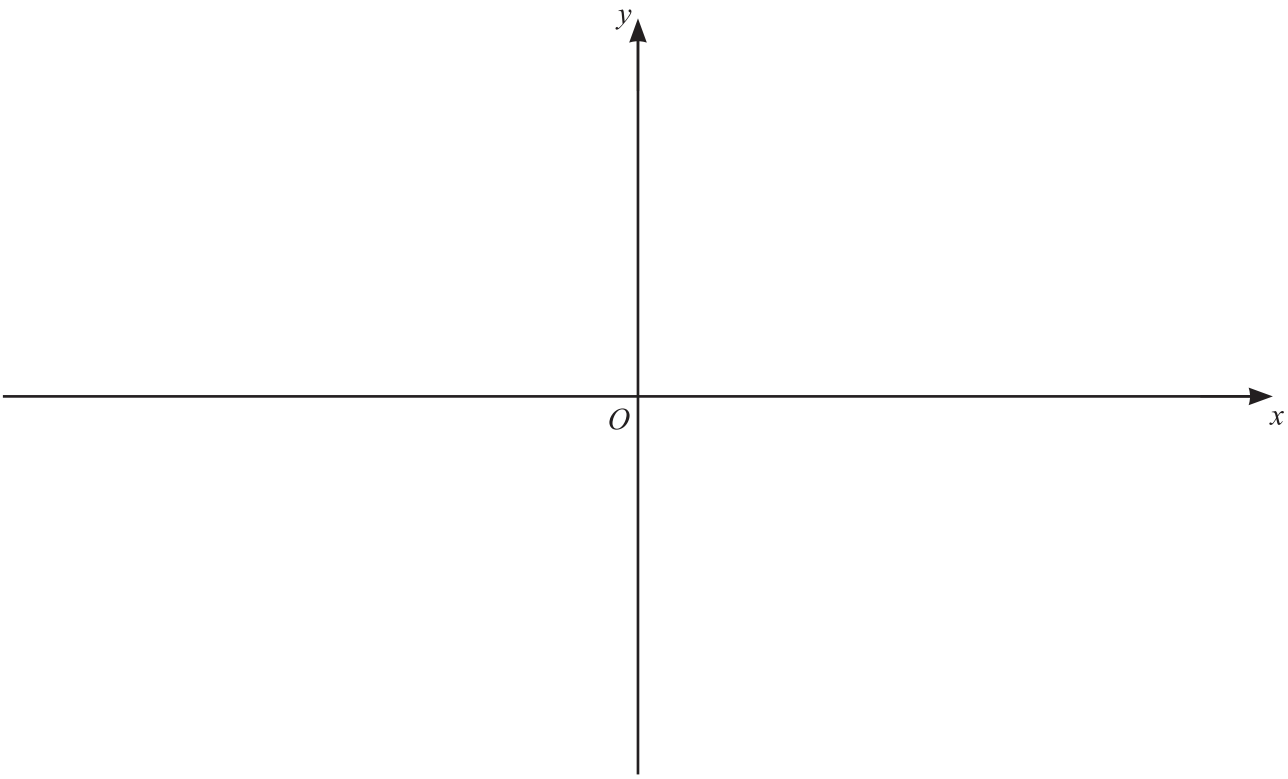
(iii) Find the number of committees that can be chosen if there has to be at least one dentist on the committee. [2]

8 **(a)** It is given that $\mathrm{f}: x \to (3x+1)^2 - 4$ for $x \geqslant a$, and that f^{-1} exists.

(i) Find the least possible value of a. [1]

(ii) Using this value of a, write down the range of f. [1]

(iii) Using this value of a, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the axes, stating the intercepts with the coordinate axes. [4]

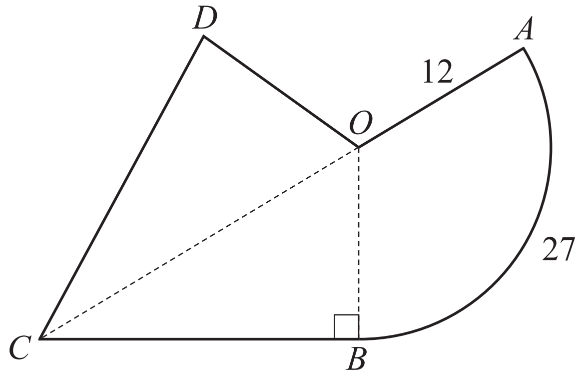


(b) It is given that
$$g(x) = \ln(2x^2 + 5) \quad \text{for } x \geqslant 0,$$
$$h(x) = 3x - 2 \quad \text{for } x \geqslant 0.$$

Solve the equation $hg(x) = 4$ giving your answer in exact form. [3]

9 Solve the equation $12x^{\frac{2}{3}} - 5x^{-\frac{2}{3}} - 11 = 0$ for $x > 0$. Give your answer correct to one decimal place. [4]

10 In this question all lengths are in centimetres and all angles are in radians.

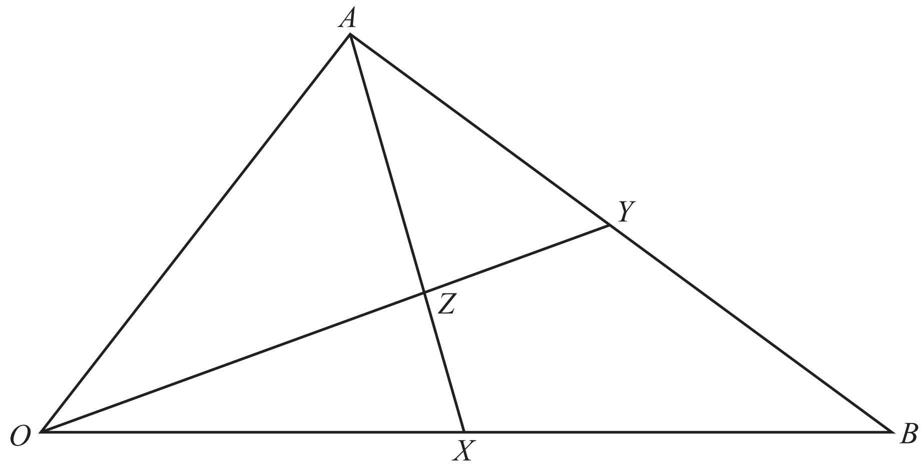


The diagram shows a badge which consists of a minor sector, OAB, of the circle with centre O and radius 12, and a kite $OBCD$, where $OB = OD$ and $CD = CB$. The arc AB has length 27. The line OB is perpendicular to the line CB, and COA is a straight line.

(a) Find the perimeter of the badge. [4]

(b) Find the area of the badge. [3]

11



In the triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The mid-point of the line OB is X, and the mid-point of the line AB is Y. The lines OY and AX intersect at the point Z. It is given that $\overrightarrow{AZ} = \lambda\overrightarrow{AX}$ and $\overrightarrow{OZ} = \mu\overrightarrow{OY}$ where λ and μ are rational numbers.

(a) Find $\overrightarrow{OZ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ. [3]

(b) Find $\overrightarrow{OZ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and μ. [2]

(c) Find the values of λ and μ. [3]

(d) Hence find $\overrightarrow{OZ}$ in terms of **a** and **b** only. [1]

Question 12 is printed on the next page.

12 A curve has equation $y = \dfrac{\sqrt{5x-2}}{x-3}$.

(a) Explain why the curve does not exist when $x < \dfrac{2}{5}$. [1]

(b) Show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{-(Ax+B)}{2(x-3)^2\sqrt{5x-2}}$, where A and B are positive integers. [5]