Cambridge International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

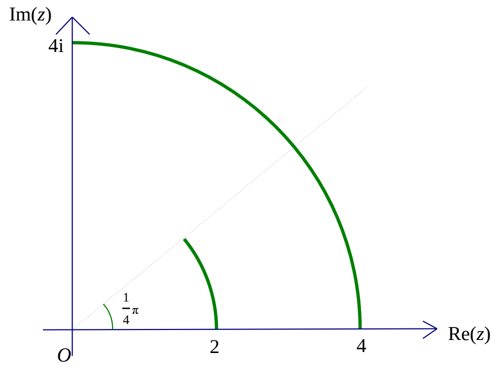
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	For all 4 marks, scales must be approximately equal, dashes can replace numbers. Arcs don't have to be perfectly circular, mark intention.		
	Show an arc of a circle, centre the origin and radius 2. Only need 2 on Re(z) or 2i on Im(z) or r = 2 to show correct radius	**B1**	
	Show an arc centre the origin for $0 \leqslant \arg z \leqslant \frac{1}{4}\pi$ with any radius	**B1**	
	Max B1 if sector shaded		
		2	
1(b)	Show an arc of a circle, centre the origin and radius 4. Only need 4 on Re(z) or 4i on Im(z) or r = 4 to show correct radius	**B1**	
	Show an arc centre the origin for $0 \leqslant \arg z \leqslant \frac{1}{2}\pi$ with any radius	**B1**	
	Max B1 if sector shaded		
		2	

Question	Answer	Marks	Guidance
2(a)	State or imply the equation $x = \sqrt{\frac{4}{5-2x}}$ and square the equation	***B1**	Could work with x_{n+1} throughout or with x_n throughout instead of x.
	Rearrange this with at least one intermediate step in the form $2x^3 - 5x^2 + 4 = 0$	**DB1**	
	Alternative Method 1 for Question 2(a)		
	Rearrange $2x^3 - 5x^2 + 4 = 0$ to $x^2(5 - 2x) = 4$ (or a different intermediate step) and to either $x^2 = \frac{4}{5-2x}$ or $x = \sqrt{\frac{4}{5-2x}}$ and then obtain the iterative formula $x_{n+1} = \sqrt{\frac{4}{5-2x_n}}$	**B2**	
	Alternative Method 2 for Question 2(a)		
	Rearrange $2x^3 - 5x^2 + 4 = 0$ to $x^2(5 - 2x) = 4$ (or a different intermediate step) and to $x^2 = \frac{4}{5-2x}$ and to $x_{n+1}{}^2 = \frac{4}{5-2x_n}$	***B1**	Must have introduced x_{n+1} and x_n.
	Obtain the iterative formula $x_{n+1} = \sqrt{\frac{4}{5-2x_n}}$	**DB1**	
		2	

Question	Answer	Marks	Guidance
2(b)	Use the iterative process correctly at least once	**M1**	The question specifies initial value 1.2, so must use the formula to obtain a value and then use this value in the formula.
	Obtain final answer 1.28	**A1**	Can gain this mark even if less than 4 dp shown in iteration.
	Show sufficient iterations to at least 4 dp to justify 1.28 to 2 dp or show that there is a sign change in the interval $(1.275, 1.285)$	**A1**	$1.2,\ 1.2403,\ 1.2601,\ 1.2700, 1.2752, 1.2778, \ldots$ Allow small errors, truncation and recovery.
		3	

Question	Answer	Marks	Guidance
3(a)	State or imply that $\ln P = \ln a + kt$ or ln P = ln a + k(ln e)t $\ln P = \frac{1}{20}t + 3$ B0 until associated with a and /or k	**B1**	Can be implied by both a and k correct. $P = e^3 e^{\frac{1}{20}t}$ gets B1B1.
	State $k = \frac{1}{20}$, not from $\frac{dP}{dt} = k$	**B1**	OE. Can be embedded in $P = ae^{kt}$.
	$\ln a = 3 \Rightarrow a = 20$ to 2 sf	**B1**	Must be 2 sf, can be embedded in $P = ae^{kt}$.
		3	
3(b)	Form a correct equation in t using a and k, or *their* a and k where a will cancel (or are both numerical)	**M1**	E.g. $2a = ae^{kt}$, $2 = e^{kt}$, kt = ln 2.
	Obtain $t = 14$[hours]	**A1**	Allow 13.75 [hrs] (13 hrs 45 min) to 14 [hrs]. ISW
		2	

Question	Answer	Marks	Guidance
4	Substitute $z = x + \mathrm{i}y$ and obtain a horizontal equation Do not allow if this would lead to an equation containing xy terms which do not cancel	*M1	E.g. $5\big(x+(y-3)\mathrm{i}\big) = (2-9\mathrm{i})\big(x+(y+3)\mathrm{i}\big)$
	Use $\mathrm{i}^2 = -1$ anywhere	M1	
	Obtain e.g. $5x + 5(y-3)\mathrm{i} = (2x+9y+27) + \mathrm{i}(2y+6-9x)$ or e.g. $3x^2 + 3y^2 - 12y - 63 + (9x^2 + 9y^2 - 30x + 54y + 81)\mathrm{i} = 0$	A1	Or equivalent expression free of products of complex numbers. Terms can be in any order.
	Obtain simultaneous equations by equating real and imaginary parts	DM1	E.g. $3x - 9y = 27$ and $3y + 9x = 21$ $3x^2 + 3y^2 - 12y - 63 = 0$ and $9x^2 + 9y^2 - 30x + 54y + 81 = 0$
	Obtain $[z =]\ 3 - 2\mathrm{i}$ only	A1	

Question	Answer	Marks	Guidance
4	**Alternative Method for Question 4:**		
	Obtain a horizontal equation in z Do not allow if it would lead to an equation containing z^2 where the xy terms do not cancel	***M1**	E.g. $5z - 15i = 2z + 6i - 27i^2 - 9iz$. Allow errors, but no brackets.
	Use $i^2 = -1$ anywhere	**M1**	
	Obtain $z = \frac{9+7i}{1+3i}$	**A1**	OE (might have an uncancelled factor of 3)
	Multiply top and bottom by $1-3i$ or equivalent for *their z*	**DM1**	Must see working for numerator or denominator, e.g. $9-27i+21+7i$ or 1 + 9 or 10. If $\frac{9+7i}{1+3i} = 3-2i$ M0A0. If $\frac{9+7i}{1+3i} \times \frac{1-3i}{1-3i} = 3-2i$ M0A0 **SC B1**. If $\frac{9+7i}{1+3i} \times \frac{1-3i}{1-3i}$ and working in numerator or denominator and $3-2i$ M1A1.
	Obtain $[z =]\ 3-2i$ only	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Rewrite $\cos^4\theta$ as $\left(\frac{1+\cos 2\theta}{2}\right)^2$ or $\sin^4\theta$ as $\left(\frac{1-\cos 2\theta}{2}\right)^2$ or $4\sin^2\theta\cos^2\theta$ as $\sin^2 2\theta$	**B1**	Starting on left. Double angle for one term.
	Obtain $\left(\frac{1+\cos 2\theta}{2}\right)^2 - \left(\frac{1-\cos 2\theta}{2}\right)^2 - \sin^2 2\theta$	**B1**	OE, e.g. $1\times\cos 2\theta - \sin^2 2\theta$.
	Expand to $\frac{1}{4}+\frac{1}{2}\cos 2\theta+\frac{1}{4}\cos^2 2\theta-\left(\frac{1}{4}-\frac{1}{2}\cos 2\theta+\frac{1}{4}\cos^2 2\theta\right)-(1-\cos^2 2\theta)$ and simplify to obtain $\cos^2 2\theta+\cos 2\theta-1$	**B1**	AG
	Alternative Method 1 for Question 5(a):		
	Express $\cos^4\theta-\sin^4\theta$ as $\left(\cos^2\theta+\sin^2\theta\right)\left(\cos^2\theta-\sin^2\theta\right)$ or rewrite $4\sin^2\theta\cos^2\theta$ as $\sin^2 2\theta$	**B1**	Starting on left.
	Simplify to $\cos 2\theta-\sin^2 2\theta$	**B1**	If $\cos^4\theta-\sin^4\theta=\cos 2\theta$ instead of $\left(\cos^2\theta+\sin^2\theta\right)\left(\cos^2\theta-\sin^2\theta\right)=\cos 2\theta$, B0.
	Use $\sin^2 2\theta=1-\cos^2 2\theta$ to obtain $\cos^2 2\theta+\cos 2\theta-1$	**B1**	AG

Question	Answer	Marks	Guidance
5(a)	**Alternative Method 2 for Question 5(a):**		
	Use correct double angle formulae once e.g. replace $\cos 2\theta$ with $\cos^2\theta - \sin^2\theta$ $\left(\cos^2\theta - \sin^2\theta\right)^2 + \left(\cos^2\theta - \sin^2\theta\right) - 1$	**B1**	Starting on right. Double angle for one term.
	Expand to obtain $\cos^4\theta - 2\sin^2\theta\cos^2\theta \boldsymbol{- \sin^4\theta + 2\sin^4\theta} + \cos^2\theta - \sin^2\theta - 1$ * or $\cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta + \cos^2\theta - \sin^2\theta - 1$ leading to $\cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta - 2\sin^2\theta$ leading to $\cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta\ \boldsymbol{-2\sin^2\theta\left(\cos^2\theta + \sin^2\theta\right)}$ **	**B1**	Write $\sin^4\theta$ as $-\sin^4\theta + 2\sin^4\theta$. Write $2\sin^2\theta$ as $2\sin^2\theta\,(\cos^2\theta + \sin^2\theta)$.
	Rewrite as * $\cos^4\theta - 2\sin^2\theta\cos^2\theta - \sin^4\theta + 2\sin^4\theta - 2\sin^2\theta$ leading to $\cos^4\theta - 2\sin^2\theta\cos^2\theta - \sin^4\theta + 2\sin^2\theta\left(\sin^2\theta - 1\right)$ leading to $\cos^4\theta - 4\sin^2\theta\cos^2\theta - \sin^4\theta$ ** $\cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta - 2\sin^2\theta\cos^2\theta - 2\sin^4\theta$ leading to $\cos^4\theta - 4\sin^2\theta\cos^2\theta - \sin^4\theta$	**B1**	
		3	

Question	Answer	Marks	Guidance
5(b)	State a quadratic equation in $\cos 2\alpha$ and solve for α $\left(\cos^2 2\alpha + \cos 2\alpha - 1 = 0\right)$	**M1**	Alternative: form a quadratic in $\tan^2 \alpha$ and solve for α $\left(\tan^4 \alpha + 4\tan^2 \alpha - 1 = 0\right)$.
	Obtain $\alpha = 25.9°$ or $\alpha = 154.1°$	**A1**	May be more accurate. Allow 154 for 154.1.
	Obtain $\alpha = 25.9°$ and $\alpha = 154.1°$ and no others in range	**A1**	May be more accurate. Allow 154 for 154.1. Mark answers in radians as a misread (0.452, 2.69).
		3	

Question	Answer	Marks	Guidance
6(a)	$P(2,\ 1, -3)$	**B1**	Accept $x = 2$, $y = 1$, $z = -3$. Do not accept 2**i** + **j** – 3**k** or $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.
		1	

Question	Answer	Marks	Guidance
6(b)	Use the correct method to find the scalar product of the direction vectors	**M1**	$\pm(-1\times 2+2\times 5)=\pm 8$ Allow error of 0 × –1 = –1.
	Divide the scalar product by the product of the moduli to obtain $\pm\cos\theta$ using consistent vectors throughout	**M1**	$\dfrac{\textit{their}\ 8}{\sqrt{\textit{their}\ 5}\sqrt{\textit{their}\ 30}}$
	Obtain $\cos\theta=\dfrac{8}{5\sqrt{6}}$	**A1**	OE, e.g. $\dfrac{8}{\sqrt{150}}$ or $\dfrac{4\sqrt{6}}{15}$. If no $\dfrac{8}{5\sqrt{6}}$ seen, just 49.2, then A0. Decimal only seen, A0. ISW
6(b)	**Alternative Method for Question 6(b):**		
	Use of cosine rule: e.g. sides of $\sqrt{5},\sqrt{30}$ and $\sqrt{19}$ found	**B1**	Could use other points.
	e.g. $\cos\theta=\dfrac{5+30-19}{2\sqrt{5}\sqrt{30}}$	**M1**	
	Obtain $\cos\theta=\dfrac{8}{5\sqrt{6}}$	**A1**	OE, e.g. $\dfrac{8}{\sqrt{150}}$ or $\dfrac{4\sqrt{6}}{15}$ or $\dfrac{8}{\sqrt{5}\sqrt{30}}$. If no $\dfrac{8}{5\sqrt{6}}$ seen, just 49.2, then A0. Decimal only seen, A0. ISW
		3	

Question	Answer	Marks	Guidance
6(c)	Any two of $\lvert PA\rvert = 2\sqrt{5}$ $\lvert PB\rvert = \sqrt{30}$ or $\lvert AB\rvert = \sqrt{82}$ seen	**B1**	May be seen by stating or implying that $\lambda = 2$ and $\mu = -1$.
	Area $= \frac{1}{2} \times 2\sqrt{5} \times \sqrt{30} \times \sqrt{1 - \frac{64}{150}}$	**M1**	Correct method for the exact area of the triangle. Note that: sin APB = $\frac{\sqrt{129}}{15}$ sin ABP = $\sqrt{\frac{86}{615}}$ cos ABP = $\frac{46}{\sqrt{2460}}$ Perp A to BP = $\frac{\sqrt{2580}}{15}$ Perp B to AP = $\frac{\sqrt{430}}{5}$
	$= \sqrt{86}$	**A1**	Or simplified exact equivalent. ISW
	Alternative Method for Question 6(c)		
	$\overrightarrow{PA} \times \overrightarrow{PB} = -4\mathrm{i} - 18\mathrm{j} - 2\mathrm{k}$	**B1**	$\overrightarrow{PA} = -2\mathrm{i} + 4\mathrm{k}$, $\overrightarrow{PB} = -2\mathrm{i} + \mathrm{j} - 5\mathrm{k}$.
	Area = $\frac{1}{2}\left\lvert \overrightarrow{PA} \times \overrightarrow{PB} \right\rvert$ = $\frac{1}{2}\sqrt{16 + 324 + 4}$	**M1**	Correct method for the exact area of the triangle.
	$= \sqrt{86}$	**A1**	Or simplified exact equivalent. ISW
		3	

Question	Answer	Marks	Guidance
7(a)	Obtain $\frac{dx}{dt} = 6\cos 2t$	**B1**	Allow $3.2\cos 2t$. dx = 6cos2t is B0.
	Obtain $\frac{dy}{dt} = \sec^2 t - \operatorname{cosec}^2 t$	**B1**	Any equivalent form. dy = $\sec^2 t - \operatorname{cosec}^2 t$ is B0, but $\frac{dy}{dx} = \frac{\sec^2 t - \operatorname{cosec}^2 t}{6\cos 2t}$ can go on to gain M1M1A1, so 3/5 possible.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	***M1**	$\frac{dy}{dx} = \frac{\sec^2 t - \operatorname{cosec}^2 t}{6\cos 2t}$
	Express as a single fraction With $\frac{dy}{dt}$ correctly simplified as a single fraction in terms of sin t and cos t Allow with 6cos 2t expressed as $\frac{1}{6\cos 2t}$ outside bracket	**DM1**	Allow $\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t} \times \frac{1}{6\cos 2t}$ or $\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t} \div 6\cos 2t$ or $\frac{\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t}}{6\cos 2t}$
	Obtain $\left(\frac{-4\cos 2t}{6\cos 2t \times \sin^2 2t} =\right)\frac{-2}{3\sin^2 2t}$ from full and correct working Numerator and denominator must have identical terms before cancelling, both cos2t or both + and – ($\sin^2 t - \cos^2 t$)	**A1**	AG Allow slips in θ and x for t to recover earlier marks, provided these are corrected before the final line. Do not allow serious errors in working for the final mark.

Question	Answer	Marks	Guidance
7(a)	**Alternative Method for Question 7(a):**		
	$y=\frac{6}{x}$	**B2**	Using $y=\frac{\tan^2 t+1}{\tan t}=\frac{\sec^2 t}{\frac{\sin t}{\cos t}}=\frac{1}{\sin t\cos t}$
	$\frac{dy}{dx}=-6x^{-2}$	**B1**	
	$\frac{dy}{dx}=-\frac{6}{9\sin^2 2t}$	**M1**	
	Obtain $\frac{dy}{dx}=\frac{-2}{3\sin^2 2t}$ from full and correct working	**A1**	
		5	
7(b)	Gradient of normal = $\frac{3}{2}$	**B1**	
	Use correct method to find the equation of the normal	**M1**	E.g. $(y-2)=\frac{3}{2}(x-3)$ or find c in $y=\frac{3}{2}x+c$. Allow a wrong value for x or y but not both, with *their* normal gradient.
	Obtain $2y-3x+5=0$	**A1**	Or $k(2y-3x+5)=0$, where k is an integer.
		3	

Question	Answer	Marks	Guidance
8(a)	State or imply the form $\frac{A}{a-2x}+\frac{B}{3a+x}$ and use a correct method to find a constant	**M1**	
	Obtain $A=2a$ or $B=a$	**A1**	
	Obtain $A=2a$ and $B=a$	**A1**	
		3	
8(b)	Use a correct method to obtain the first two terms of the expansion of $(a-2x)^{-1}$ or $\left(1-\frac{2x}{a}\right)^{-1}$ or $(3a+x)^{-1}$ or $\left(1+\frac{x}{3a}\right)^{-1}$	**M1**	
	Obtain $+2\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+...\right)$	**A1ft**	OE. May be unsimplified. Follow *their* A, B for an expansion involving a.
	Obtain $+\frac{1}{3}\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+...\right)$	**A1ft**	OE. May be unsimplified. Follow *their* A, B for an expansion involving a.
	Obtain $+\frac{7}{3}+\frac{35x}{9a}+\frac{217x^2}{27a^2}$	**A1**	Or simplified equivalent. Final answer. Ignore any terms in higher powers of x. Do not ISW, e.g. multiplying by $27a^2$. Condone different order of terms.

Question	Answer	Marks	Guidance
8(b)	**Alternative Method for Question 8(b)**		
	Expanding $7a^2(a-2x)^{-1}(3a+x)^{-1}$ from the original question. Use a correct method to obtain the first two terms of the expansion of $(a-2x)^{-1}$ or $\left(1-\frac{2x}{a}\right)^{-1}$ or $(3a+x)^{-1}$ or $\left(1+\frac{x}{3a}\right)^{-1}$	**M1**	
	Obtain $+7a\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+...\right)$ or $+\frac{7a}{3}\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+...\right)$	**A1**	OE. May be unsimplified. May be implied by the expression shown for the next A1.
	Obtain $+\frac{7}{3}\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+...\right)\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+...\right)$	**A1**	OE. May be unsimplified.
	Obtain $+\frac{7}{3}+\frac{35x}{9a}+\frac{217x^2}{27a^2}$	**A1**	Or simplified equivalent. Final answer. Ignore any terms in higher powers of x. Do not ISW, e.g. multiplying by $27a^2$. Condone different order of terms.
		4	
8(c)	$\|x\|<\frac{a}{2}$	**B1**	Or $-\frac{a}{2}<x<\frac{a}{2}$. Mark final answer. Must make a clear statement.
		1	

Question	Answer	Marks	Guidance
9(a)	Divide to obtain quotient x^2+k	**M1**	k is a constant.
	Obtain quotient x^2-4	**A1**	If quotient stated separately, mark at this stage.
	Obtain remainder 32	**A1**	If remainder stated separately, mark at this stage. Need not state which is quotient and remainder, but if stated wrongly, max 2/3. After a correct division, still allow the marks if then written as $x^2-4+\dfrac{32}{x^2+4}$.
	Alternative Method for Question 9(a)		
	Expands brackets to get $B = 0$	**M1**	$(x^2+4)(x^2+Bx+C)+D=$ $x^4+Bx^3+(C+4)x^2+4Bx+4C+D$
	$C = -4$	**A1**	
	$D = 32$	**A1**	Need not state which is quotient and remainder, but if stated wrongly, max 2/3.
		3	

Question	Answer	Marks	Guidance
9(b)	$\frac{1}{3}x^3 - 4x$	**B1 FT**	Follow *their* quotient of form $Ax^2 + B$.
	Obtain $p\tan^{-1}qx$ where $q = 2$ or $q = \frac{1}{2}$	**M1**	
	Obtain $16\tan^{-1}\frac{1}{2}x$	**A1 FT**	Follow *their* constant remainder, i.e. $\left(\frac{\textit{their}\text{ constant remainder}}{2}\right)\tan^{-1}\frac{1}{2}x$.
	Use limits correctly in an expression containing $p\tan^{-1}qx$ where $q = 2$ or $q = \frac{1}{2}$ and $rx^3 + sx$	**M1**	Terms need not be evaluated, e.g. $\left[8\sqrt{3} - 8\sqrt{3}\right] + 16\tan^{-1}\sqrt{3} - \left(\frac{8}{3} - 8 + 16\tan^{-1}1\right)$ or $\frac{8}{3} - 8$ can be $-\frac{16}{3}$, $16\tan^{-1}\sqrt{3}$ can be $\frac{16\pi}{3}$, $16\tan^{-1}1$ can be 4π.
	Obtain $\frac{4}{3}(\pi + 4)$ from full and correct working	**A1**	AG
		5	

Question	Answer	Marks	Guidance
10(a)	State that $\frac{dV}{dt} = 50\,000 - 600h$	**B1**	May be seen as $\frac{dV}{dt} = 50\,000$ and $\frac{dV}{dt} = [-]600h$. When put together (may be in the chain rule) B1 can be awarded.
	[Use $V = 40\,000h$ to] obtain $\frac{dV}{dh} = 40\,000$ and use this and *their* $\frac{dV}{dt}$ in the correct chain rule to obtain $\frac{dh}{dt}$ or [Use $V = 40\,000h$ to] obtain $\frac{dV}{dt} = 40\,000\frac{dh}{dt}$ and equate to *their* $\frac{dV}{dt}$	**M1**	$\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$ E.g. $\frac{50\,000 - 600h}{40\,000} = \frac{dh}{dt}$ E.g. $40\,000\frac{dh}{dt} = 50\,000 - 600h$

Question	Answer	Marks	Guidance
10(a)	Obtain $200\dfrac{dh}{dt}=250-3h$ from full and correct working	**A1**	AG $\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{50000-600h}{40000}$ OE, leading to given answer with no other working, or no incorrect working seen **SC B1** (1/3). $\dfrac{\mathrm{d}V}{\mathrm{d}t}=50000-600h$ B1 followed by $\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{50000-600h}{40000}$ OE, leading to given answer with no other working, or no incorrect working seen, **SC B1** 2/3. $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}h}\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$ followed by $50000-600h=40000\dfrac{\mathrm{d}h}{\mathrm{d}t}$ OE, leading to given answer with no other working, or no incorrect working seen, B1 for implied $\dfrac{\mathrm{d}V}{\mathrm{d}t}$ and **SC B1** 2/3.
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Question	Answer	Marks	Guidance
10(b)	Separate variables correctly and integrate one side correctly	**M1**	E.g. $\int \frac{1}{250-3h}\mathrm{d}h = \int \frac{1}{200}\mathrm{d}t$. Integral signs may be omitted, 200 may be on opposite side.
	Obtain $-\frac{1}{3}\ln\lvert 250-3h\rvert = \frac{t}{200}\left(+C\right)$	**A1**	OE Condone missing "+C" and lack of modulus signs.
	Use $t=0,\ h=50$ in an expression containing $\ln\left(250-3h\right)$ or $\ln\lvert 250-3h\rvert$ to find the constant of integration.	**M1**	Or equivalent use of limits 50 and 80.
	Obtain $C = -\frac{1}{3}\ln 100$	**A1**	OE, e.g. $\frac{1}{3}\ln\frac{100}{250-3h} = \frac{t}{200}$, or $-\frac{200}{3}\ln\left(\frac{10}{100}\right)$. With or without modulus signs on the log terms.
	$t = 150$	**A1**	
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Question	Answer	Marks	Guidance
11(a)	Use of correct product rule and correct chain rule	**M1**	$\frac{dy}{dx} = A\cos x\sqrt{2+\cos x} + \frac{B\sin x\sin x}{\sqrt{2+\cos x}}$
	Obtain $\frac{dy}{dx} = 2\cos x\sqrt{2+\cos x} - \frac{2\sin^2 x}{2\sqrt{2+\cos x}}$	**A1**	OE
	Equate the derivative to zero and obtain a horizontal 3 term quadratic equation or 4 term quartic equation in $\cos a$ If M0 earlier then needs that expression to be such that arrive at 3 term quadratic or 4 term quartic equation in cos x without further trig errors. The only error in the form of the differential allowed is for $(2+\cos x)^{-\frac{1}{2}}$ to be $(2+\cos x)^{+\frac{1}{2}}$ or $(2+\cos x)^{-\frac{3}{2}}$	***M1**	Accept in cos x. E.g. $3\cos^2 x + 4\cos x - 1 = 0$. E.g. $3\cos^4 x + 16\cos^3 x + 18\cos^2 x - 1 = 0$.
	Solve for cos a	**DM1**	$\left(\cos a = \frac{-2+\sqrt{7}}{3} \text{ or } 0.215\right)$ Allow presence of other solution(s).
	Obtain $a = 4.93$	**A1**	Allow more accurate, e.g. 4.929… even though question states 2 dp. If x = 1.35 leads to x = 4.93 award A1 BOD. If x = 1.35 and x = 4.93 award A0.
		5	

Question	Answer	Marks	Guidance
11(b)	State or imply $\mathrm{d}u = -\sin x\,\mathrm{d}x$	**B1**	OE If B0, max M1M1M1.
	Substitute throughout for u and du	**M1**	
	Obtain $-\int 2\sqrt{u}du$	**A1**	OE. Ignore limits if $-\int 2\sqrt{u}du$, but if $+\int 2\sqrt{u}du$, then must have correct limits $\int_1^3 2\sqrt{u}\mathrm{d}u$. (See final M1)
	Integrate to obtain $ku^{\frac{3}{2}}(+C)$	**M1**	Constant of integration not required
	Use correct limits correctly in an expression of the form $ku^{\frac{3}{2}}$ or $k(2+\cos x)^{\frac{3}{2}}$	**M1**	1 and 3 for u, or 0 and π for x.
	Obtain $\frac{4}{3}\left(3\sqrt{3}-1\right)$ or $4\sqrt{3}-\frac{4}{3}$ or $\frac{4}{3}\sqrt{27}-\frac{4}{3}$	**A1**	OE. Allow, e.g., $\sqrt{3}^3$ for $\sqrt{27}$. ISW but don't ignore e.g. multiplying throughout by 3. If the answer is changed from negative to positive value at end, then A0. Last M1A1 can use modulus, providing no errors seen.
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